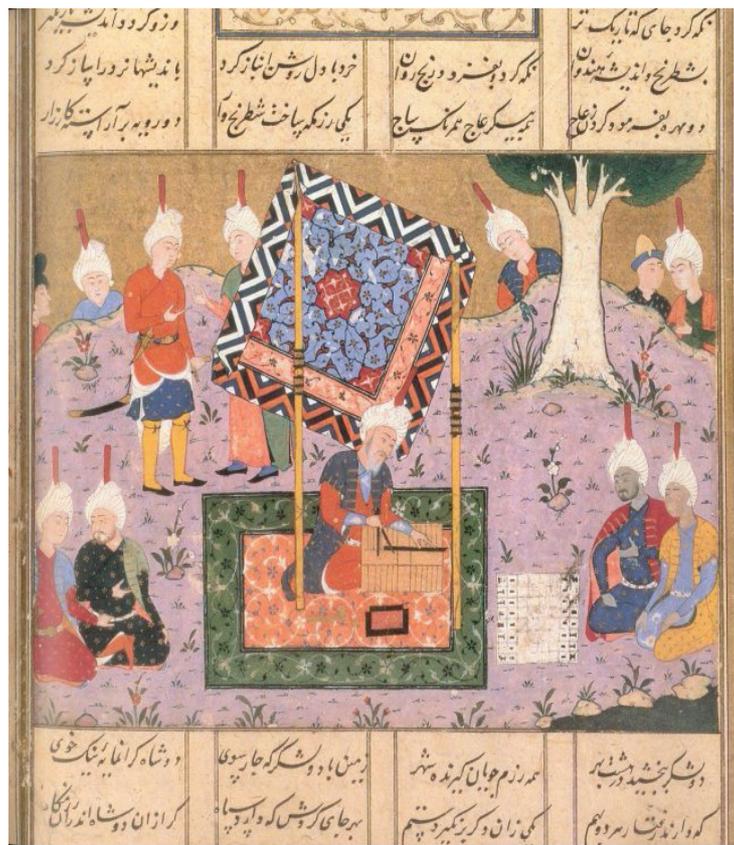


A game, a challenge, an enigma

We are currently confronted with the opinion, spread by the media, that we are living the consequences of a clash of cultures. Following this view, it seems inevitable that two different ways of seeing the world should lead to a conflict. That only repression and social convenience could quell the constant friction between them.

History, on the other hand, provides us a broad quantity of examples where different cultures could meet without ending in violent confrontations, nor in a mutual annihilation in a mix, but that led to a mutual enrichment. But we should highlight that for this to happen, the encounter should take place to a deeper level than the one of a shallow tolerant coexistence. An active willingness to take contact and exchange should be present.

This study, analysing the example of an encounter between Indian and Persian cultures, wants to show an example of how this can happen. This is the story of an intellectual challenge, not a military one. Two board games are exchanged and constitute the field where the superiority of one culture must be measured. Coming from a Persian source, the end of the story obviously celebrates the victory of this culture. But, as we will see, the symbolism of the tale shows the communication of the two cultures at a deeper level, demonstrating the existence, at the basis, of a shared way of thinking and feeling.



The legend of the Nard's invention at the Persian court in response to the arrival of the chess from India, as depicted in the Muslim world. (From <http://history.chess.free.fr>)

A Persian tale from the sixth century

Throughout the ages, chess has fascinated people, and not only chess players but also laymen. The squared floor that hosts the movements of hieratic pawns obeying strict rules is the source of innumerable metaphors regarding human life, the struggle between order and chaos, destiny and free will, knowledge and unpredictability.

The obscure origin of this game contributes further to its charm. Even though nowadays we can count on a huge bibliography on the argument, comprehending the work of great scholars like Joseph Needham and Paul Thieme, the origin and the symbolic meaning of chess has vanished in the mists of time.

And yet investigating the origins of this game we bring to light a rich symbolism that confirms and amplifies its enigmatic charm.

The intention of this brief study is to take a journey in the depths of time to unravel some mysteries that the board and its enigmatic pieces treasure.

The key to open up the door and begin our journey is found in a Persian manuscript. A tale from the sixth century A.D. that connects the origin of chess with the invention of backgammon. Its title is "The explanation of chess and the invention of backgammon".¹

In this tale an Indian king sends a wagon loaded with precious gifts to the Persian king Xusraw I (531-579). In amongst the riches there is a chess board and the related game pieces. It's a challenge: can the Persian sages understand the game's rules without asking someone for help? Chess, or more precisely a sort of chatrang (proto-chess) would be, according to this tale, the creation of Indian sages.

If the wise men from Persia were unable to unravel the rules of the game, the Indian king could claim the honour of the intellectual and cultural superiority of his kingdom.

A wise scholar from the court of Xusraw accepts the challenge and requires three days in order to solve the riddle. His name is Wuzurgmihr and he will not only discover the rules of the game, but also challenge in turn the sage that leads the Indian delegation, Tataritos, beating him three times. At this point, Wuzurgmihr asks the permission of King Xusraw to send a counter-delegation to India in order to challenge back the Indian sages. The object of the challenge will be a backgammon set, invented by Wuzurgmihr himself, that enemy scholars have to decipher in the same fashion as he did with chess.

The Indian king asked for forty days time, but his sages could not solve the riddle and the Persian wise was able to return home carrying a great honour and obviously the glory.

So, let us make some annotations regarding this mythical tale:

1- It is one of the first written accounts of chess and backgammon.

2- The intellectual-chivalirous challenge between Persian and Indian culture is carried out through the medium of two board games. The first represents reason and free will, the second chaos and destiny.

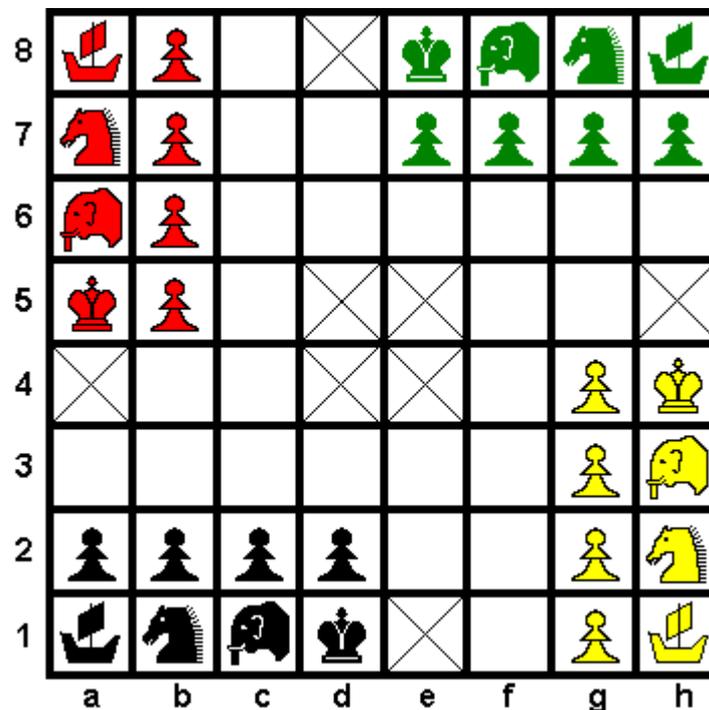
3 – The text accurately describes the astrological symbology of backgammon and of the dice used to play it, but does not mention chess symbolism at all, merely telling us that it mirrors the structure of a battlefield and of the armies that fight on it.

Taking into account all of these aspects, a fundamental question arises: how could Wuzurgmihr unravel chess rules, by only having at his disposal a chess board and the game pieces?

Assuming that we are analysing a mythical tale, which means that the episode of the Persian wise

¹ The transliteration of the original Pahlavi title is "*Wizarisn i cotrang ud nihisn i new-ardaxsir*" (WCN).

men in front of the silent square board, trying to unravel his secret, never took place in the real world, it seems that the curious elliptical and elusive nature of the narration invites us to ask ourselves if this could be actually possible. And, as we will see, this paradoxical question is the key for gaining a deeper comprehension of the myth.



The Chaturanga, Indian forerunner of chess. It was a „race game” and was played with dice. Please note the quadripartite structure of the game. (From <http://history.chess.free.fr>)

Games, duels and riddles

Like all fables, this one also possess a superficial meaning (the intellectual challenge between two kingdoms) and a deeper one, that leads us into the middle of a chess riddle. To take this path we should ask ourselves a paradoxical question, like the one that faces Alice on the other side of the mirror. We should, in other words, put ourselves in Wuzurgmihir's shoes. In front of us the square board and the strange pieces that we have never seen before.

It seems impossible to understand the rules of the game weighting these carved figures (the one in our version should be made of rubies and emeralds), trying to imagine their trajectories on the board. But the solution of the riddle is exactly here. Because maybe it is indeed possible.

Let's start examining the square board. The original board was not divided into black and white squares as it is today, but simply displayed lines dividing it into 64 little fields.

This is the first clue. In fact, even if such a board does not communicate any meaning to the contemporary reader, it did for the Indian and Persian sages of the sixth century.

To our Wuzurgmihir, personification of Zoroastrian knowledge², such a board would immediately

² Wuzurgmihir's "precognition", his ability to know "a priori" the rules of the game, has its peculiar symbolic value in the frame of Sassanid and Zoroastrian worldview. It represents the principal attribute of the positive divinity Ohrmazd, opposite to the retro cognition or "knowing after" typical of the wicked Ahriman. This contraposition is obviously linked also to the legitimation of Sassanid power, in turn linked to Zoroastrism.

Antonio Panaino, *La novella degli Scacchi e della Tavola Reale. Un'antica fonte orientale sui due giochi da tavoliere più diffusi nel mondo eurasiatico tra Tardoantico e Medioevo e sulla loro simbologia militare e*

recall the magic square of order eight. But what is a magic square? It's a particular disposition of numbers that allows the sum of line, column and diagonal to be constant. This sum is defined "magic constant", and can be calculated using the formula: $M = n(n^2 + 1) / 2$, where n defines the order of the square, in other words the number of the division on the side (for the chess board is eight). This is the magic key to open the door to the solution of our riddle.

But let us return to our tale. Why can we assume that Wuzurgmihr, as a scholar of his time, must have been very familiar with magic squares? We know, for example, from the "Encyclopaedia" of the Brethren of Purity, a group of Ismaili philosophers active in the ninth century in Bosra, that magic squares were correlated to the astrological planets basing of their order (squares on the side). This correspondence made them suitable to be broadly used as talismans referring to their astrological reference (for example a magic square of Venus should be employed in order to obtain love, one of Mercury for business prosperity, and so on).

It can be objected that the Brethren of Purity were active four centuries after the time we are referring to, in the contest of Islamic Iraq. This is true, but it is important to remember that their "Encyclopaedia", because of its very nature, had the goal to collect all previous knowledge, displaying it in a systematic manner. It seems reasonable, then, to assume that the techniques to construct magic squares and their symbolism, in the fashion handed down by the Brethren of Purity, represent an older tradition³.

We also know that in India, square boards played a fundamental role in architectural planning, conveying an intricate and complex symbology relating the square fields, the directions and Hindu Gods and divinities. For our purposes is enough to say that the two most important squares, the one containing 64 (8x8) and 81 (9x9) were the most important in planning a building. The square of 64, called Manduka, was used for the structure of temples and the buildings used by the priests (Brahmans). The square of 81 was used for civilian buildings or the ones dedicated to the cult by the caste of the warriors (Kshatriyas)⁴.

As we can see, the game board, for us mute and without meaning, was, on the contrary, eloquent for Wuzurgmihr. It was saying to him that as the game was related to Mercury, it was then a game entailing cunning. And that he was relating to Brahman, so this was the correct way to intend the king, like a character invested of sacral power and in fact flanked by a Vizier (the present queen is a substitution of this piece that took place with the arrival of the game in the Europe of chivalry)⁵.

astralogica., Mimesis, Milano, 1998. p. 49.

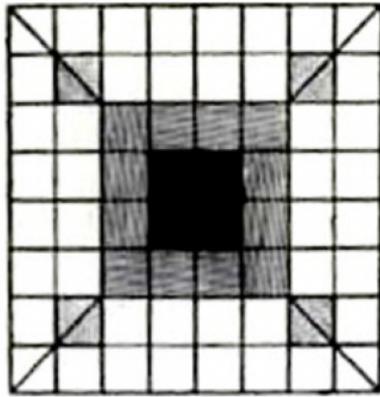
³ A tradition that could be linked to the astral cult of Harran, a city in the north of Mesopotamia, where this religion could be openly practiced even after the Islamisation of the area. It is referred to that in that city there were seven temples dedicated to the seven astrological planets, in different shapes and colours, according to the symbology of each planet. This astral philosophy, mixed to its neoplatonic equivalent, had great influence on Islamic thought contemporary to the Brethren of Purity. The "Brethren of Purity" were an Ismaili circle born in the middle of the tenth century after Christ. They published a "Universal Encyclopaedia" in 52 volumes, where, amongst other things, we can find for the first time the motto "Liberty, Equality, Fraternity."

Gabriele Mandel, *La Futuwwa, o "Cavalleria islamica"*, Lecture of prof dott Gabriele Mandel Khân, Vicarius for Italy of the sufi Jerrahi-Halveti fraternity, at the Congress on "Milizia sacra e Terzo millennio", organized in Imperia by the Accademy of Filaleti the 25th April 2005. For a deeper insight of the diffusion and the use of magic squares in Islamic context see:

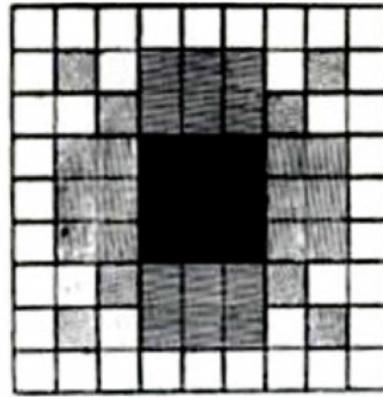
Gabriele Mandel, *La magia nell'Islam*, Simonelli, Milano, 1997. pp. 159-168.

⁴ We cannot go deeper describing the link between traditional Indian architecture, magic squares and astrological symbolism, a huge theme. To have an overview of the connection between Hindu temples and the board 8x8 see: Stella Kramrisch, *The Hindu temple*, Delhi, Moltlal Banarsidass Publishers, 1976.

⁵ We should remember that in the Sassanid court of Xusraw, the king was the bearer of a divine investiture of Zoroastrian nature. This could explain checkmate, the fact that the king cannot be simply killed, like any other piece, but must be imprisoned.



64-square mandala



81-square mandala

Divisions of the ground by 64 and 81 and their symbolism in Indian architecture. (From <http://goddesschess.blogspot.de>)

Computation boards and magic squares

Now we should briefly go back to the peculiar proprieties of magic squares. First of all we need to imagine ourselves in a completely different mathematical sensibility, where relations and quantities were perceived in a more “concrete” way. Just think about the fact that at the time Arabic numbers were still not in use and all calculation had to be computed moving little pebbles (“calculi” in Latin, origin of the word “calculation”) on square boards, those fields were marked to represent different values. In the same way, the operation of multiplying something had immediately recalled the image of a rectangle or a square (the area is obtained by multiplying side by side) and the idea of generation (the “product” of “multiplication”, offspring), and so on.

Holding this figurative way of thinking in mind, just return to Wuzurgmihr.

He observed that the square with 64 fields, like all pairs of magic squares does not have a centre, in contrast with the one with 81. This is the symbolical reason behind the correlation, in the Indian tradition, of the first with the Brahmans (spiritual power coming from the not-manifest) and the second with the Kshatriyas (power spreading from a visible centre of authority).

At this point we should take a little break, in order to account for the discovery made by some researchers, among them Ricardo Calvo, Pavle Bidev and Reinhart Wieber. They found in an Arabic manuscript (MS Berlin 7663-1), written by a certain Al-Safadi, a magic square of order 8, in which, after eight moves, every chess piece totals a sum of 260⁶.

⁶ David Shenk, *The Immortal Game: A History of Chess*, Souvenir Press Ltd, London, 2007. p.326.

8	58	59	5	4	62	63	1
49	15	14	52	53	11	10	56
41	23	22	44	45	19	18	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	47	46	20	21	43	42	24
9	55	54	12	13	51	50	16
64	2	3	61	60	6	7	57

Representation of The Safadi Board

Arabic Manuscript
MS Berlin 7663-1

8	7	59	60	61	62	2	1
16	15	51	52	53	54	10	9
41	42	22	21	20	19	47	48
33	34	30	29	28	27	39	40
25	26	38	37	36	35	31	32
17	18	46	45	44	43	23	24
56	55	11	12	13	14	50	49
64	63	3	4	5	6	58	57

Illustration of the concept of “secret centre” and reproduction of the “Safavi board”, which, according to Calvo, contains the “genetic code” of chess. (Copyright Donald McLean - 2003)

Calvo points out that this board (defined as the “Safavi board” by him), even today still bears a great intellectual momentum, because it contains what he defines as the “genetic code” of chess. Calvo demonstrates a simple method by which Wuzurgmihr could construct this board and use it to test the way each piece moves.

But this procedure is only valid as a verification, after the actual moves are already discovered. In fact, this board does not provide us any information as to which piece moves in what way. To discover this correlation there must be a numeric correlation between the structure of the magic square and the chess-piece itself. In other words, every piece should correlate to a numerical value, which can be done by ordering them hierarchically (the dimension of the pieces often helps when the figure is represented in an abstract way). This value should then be found in the board in a way that defines the correlated move.

Now, we saw that the magic sum, which is the result of adding the numbers in lines, columns and diagonals is equal to $M = n(n^2 + 1) / 2$. This sum has in turn a relation with the number in the centre of the square (when of odd order), because it is obtained by multiplying it by the number of the order of the square. For example if we take the magic square of order 3, the magic sum 15 is obtained by multiplying the central number (3) by the central number (5).

But the board in front of Wuzurgmihr has a pair number as order (8) and his 64 little squares do not have any centre.

What happens then, if I try to calculate his centre anyway? The formula being $(n^2 + 1) / 2$, we obtain $65/2$ which is not an integer. But let me just point out the 65. What is interesting about it? That opposed the squares in a symmetrical magic square⁷, when added together, should always be equal to $(n^2 + 1)$, s $(64 + 1)$. If we look at the magic square of order 3 this relationship between the centre and the sum of opposite squares results is evident. The sum of opposite pairs is always 10 (which is $n^2 + 1$), and when added to the centre (5) totals the magic sum 15.

So we obtain the number 65 as sum of opposite squares and $65/2$ as “hidden centre” of our magical square⁸.

⁷ Not every magic square is symmetric. A magic square of order 4 has 880 possible combinations, but only 48 of them are symmetrical. The magic squares present on ancient manuscripts, however, like those of the Brethren of Purity or of Agrippa, are always symmetrical. This predilection is surely linked with a symbolical motivation.

⁸ This definition was created by Ricardo Calvo. His observations regarding the diagonals as a method of construction of the squares are very interesting, as well as the concept of a “secret centre”, compatible with Indian architectural symbology, which employs the magic square 8x8. However, we don't agree with this analysis regarding the

Again, to understand the meaning of this clue, we should put ourselves into the way of thinking and feeling of the time.

In front of Wuzurgmihr there are the chess board and the pieces, which were probably carved in shape of soldiers, elephants, chariots, and so on, suggesting in the first place the idea that it was a war game⁹.

But even if the Indian wise men intended to present a harder challenge to their foe and provide him with pieces in an abstract shape (we will see that our solution remains valid even in this case), Wuzurgmihr had found in the number 65 a clear connection with the magic sum of the square of order 5. This, being the square related to Mars, which is related to warfare, show him without any doubt that the board must depict a battlefield.

At this point, one can think about this correlation being purely a product of fantasy. How we can be sure that Wuzurgmihr or a scholar of the time could follow such a pattern of thought? We will list some elements that speak for this theory:

1- This is the only case, between the “planetary” magic squares (from order 3 to 9) where the “partial sum” of a square equals the magic sum of another one.

2- The fact that the symbolism of the magic square of 5, related to the astrological planet Mars, connects the game with warfare and the tight link between the magic squares of order 8, 9 and 5 in Indian symbolism¹⁰.

3- The magic square of order 5 has 25 small divisions. If we don't count the centre 24 small squares, which are particularly apt with 6 different type of pieces, taking for granted that every piece moves towards 4 small squares ($24/6 = 4$)¹¹.

4- As with all human ideas, this theory should also be verified through a process of trial and error: Wuzurgmihr has 6 different kinds of pieces and has to find out the way they move on the board. If in the process of creation of a magic square of order 5, we can distinguish 6 different mathematical proprieties that we can logically correlate with the pieces, then the partial sum 65 was in fact the hidden key to discover the secret of the chess board.

“gnostic” symbolism that should be incorporated in the arrangement of the square.

⁹ This is the fourfold division of the Indian army in the 3th century B.C. It has therefore nothing to do with the real warfare contemporary to the creation of chess, but has a pure symbolical value, compatible with the use of the mandala 8x8, dedicated to temples and the Brahmana's caste.

¹⁰ It is evident that the multiplication of the order of those squares is 360- a very important number from an astronomical point of view. And maybe a reference to the legend, reported by Plutarch, in which Mercury (square 8x8) played with the Moon (square 9x9) using a board and dice and won the 5 days that are added to the standard Egyptian year of 360 days to match the actual tropical or solar year.

Titus Burckhardt, *Il simbolismo del gioco degli scacchi in La maschera sacra e altri saggi*, Archè, Milano, 1979, pp. 23-32.

¹¹ Even if we will see that for mathematical reasons, Knight and King take 8 squares for their moves.

Saturno = 15		
4	9	2
3	5	7
8	1	6

Giove = 34			
4	14	15	1
9	7	6	12
5	11	10	8
16	2	3	13

Marte = 65				
11	24	7	20	3
4	12	25	8	16
17	5	13	21	9
10	18	1	14	22
23	6	19	2	15

Venere = 175						
22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Sole = 111					
6	32	3	34	35	1
7	14	27	28	8	30
19	11	16	15	23	24
18	20	22	21	17	13
25	29	10	9	26	12
36	5	33	4	2	31

Mercurio = 260							
8	58	59	5	4	62	63	1
49	15	14	52	53	11	10	56
41	23	22	44	45	19	18	48
32	34	35	29	28	38	39	25
40	26	27	37	36	30	31	33
17	47	46	20	21	43	42	24
9	55	54	12	13	51	50	16
64	2	3	61	60	6	7	57

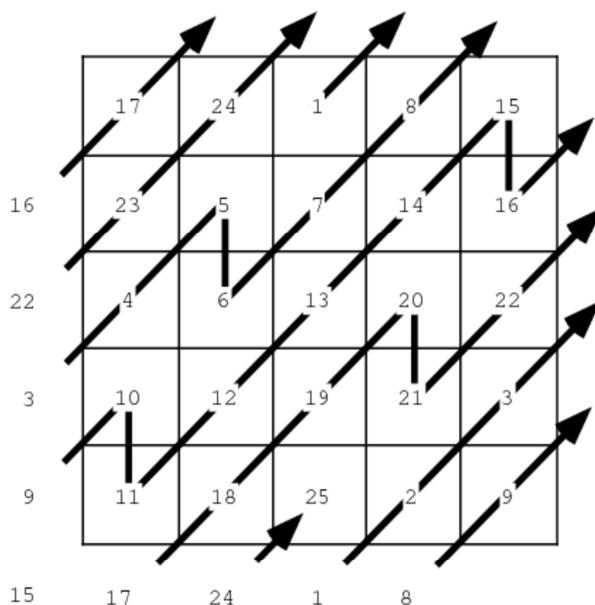
Luna = 369								
37	78	29	70	21	62	13	54	5
6	38	79	30	71	22	63	14	46
47	7	39	80	31	72	23	55	15
16	48	8	40	81	32	64	24	56
57	17	49	9	41	73	33	65	25
26	58	18	50	1	42	74	34	66
67	27	59	10	51	2	43	75	35
36	68	19	60	11	52	3	44	76
77	28	69	20	61	12	53	4	45

The "planetary" magic squares in Cornelio Agrippa's version, *De occulta philosophia*. (Da <http://www.appuntimania.com/scientifiche/matematica>)

Chessboard and dice

Not all magic squares of order 5 are equal. In fact there are 275,505,224 different possible combinations of numbers forming such a magic square.

Which one then was the one supposedly used by Wuzurgmihr? We think that he could have used the one obtained by the “Siamese method” (see below). A method illustrated by Kraitchik (1942), that seems to be the same employed by the Brethren of Purity¹².



Magic square of order 5 built using the “Siamese method”. (From Maurice Kraitchik, *Mathematical Recreations*, W.W. Norton, New York, 1942, pp. 148-149)

As we can see, this magic square is symmetrical, which means that all opposite squares add up to the same number equal to $(n^2 + 1) = 26$.

But what happens if we subtract the opposite squares in order to obtain the “value” of the different moves? Let us try¹³:

¹² (Maurice Kraitchik, *Mathematical Recreations*, W.W. Norton, New York, 1942, pp. 148-149)

This method is also called method “de la Loubere”, from the person that spread it in the Western world after having learned it in Siam where he worked as an ambassador. For its simplicity and the presence of this version in ancient sources (like Al-Gazahli), we think that this should be the magic square to which the mythical tale refers.

¹³ These moves are not compatible with modern chess moves, but with the original ones, where the Tower or Rook (which was a chariot) moved jumping the intermediate square and had a movement limited to two squares. The Bishop (which was an elephant) moved with the same limitations diagonally. The Knight had the same L move as today, whereas the pawn moved one square forward but could probably capture or take opponent’s pieces in all directions. The Queen (a general or vizier) in our reconstruction moves and captures only diagonally. It is also possible to think that it would move one square in all directions, like the King in our reconstruction. In that case, having their role switched, they would obtain the values of 12 and 16, which could also be valid.

Piece 1 : $25 - 1 = 24$ $24 - 18 = 6$ (Tower/Rook)
 $22 - 4 = 18$

Piece 2 : $17 - 9 = 8$ $8 - 4 = 4$ (Bishop)
 $15 - 11 = 4$

Piece 3 : $14 - 12 = 2$ $16 - 2 = 14$ (Queen)
 $21 - 5 = 16$

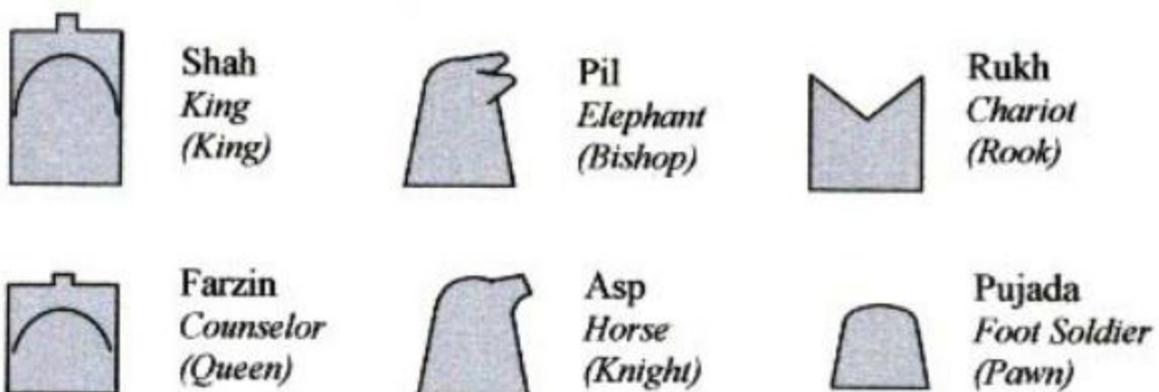
Piece 4 : $19 - 7 = 12$ $14 - 12 = 2$ (Pawn)
 $20 - 6 = 14$

At this point, we can see that the very mathematical structure of the magic square suggests that the spaces representing the moves of the knight are mirroring one another. This can be seen by observing the angles of the magic square (3-2 symmetrical to 24-23 and 18-10 mirroring 16- 8). This mathematical structure seems to suggest one single piece, whose value will be calculated following this symmetry:

Piece 5 : $8 - 1 = 7$ (Knight)

However, some doubts arise, because the last piece should move covering the squares already used by other pieces, or, conversely, two pieces moves differently as we discovered till now following a logical process.

Of the 16 pieces, 8 look the same. Their shape and dimension show clearly that they represent the least in order of importance, the soldiers on the frontline. Similarly, one piece is bigger than the others and is marked with a crown or another distinguishing feature. It is therefore the most important one. One of them must therefore assume a weak move, which could be Piece 4's, that moves only forward and back and left-right by one square, or Piece 3's, which moves one square diagonally. But this way the most important piece would be missing, and in turn, the "values" we obtained until now seem to be numerically meaningless.



Chess pieces in their stylised form, common in the Islamic world. (Da ancientchess.com)

Ages of the world and board games

The way to make sense of the numerical values obtained so far is to take a look at the Indian way of counting the spots on dice. Why dice? We know that protochess was closely linked with chaturanga and other racing games, which were played with dice. In some cases, which piece should be moved was decided by dice. The use of the dice in association with chess in Europe and Middle East is attested until the late Middle Ages.

But what is peculiar in the Indian way of counting the dice scores? This way of counting is connected with ancient Indian cosmology, where the universe is thought passing through 4 ages or Yugas, starting from the golden age, with value 4, to decay to 1 for what concerns the last and worst age, the Kali Yuga¹⁴.

A value system which finds his application also in the Mancala, a very old game. In such game two players should occupy a set of cups, filling them with small pebbles, or seeds. When 4 pebbles fill a cup, they are removed and it returns to the original vacuity or emptiness¹⁵.

This way of considering numeration is attested in chaturanga, a forerunner of chess, where dice decided which piece should move. Al-Biruni explains to us that the dice used in this game did not have the sides referring to numbers 5 and 6, and that the one moved the king or pawn, the two the chariot, the three the knight and the four the elephant¹⁶.

Bearing this in mind, we can order the series of values obtained by Wuzurgmihir starting from 4, then 6 (4 + 2) and 7 (4 + 3). The two other values 2 and 14 remain out of the series.

The solution in order to find a homogeneous series is to find a 5, a value that we can obtain considering that the weakest piece (which today is called a pawn) moves only one square forward (the weakest move) but could originally capture in every direction. The weakest piece in our values, then, the Piece 4, obtain a new value, subtracting from 7, the square immediately in front of the centre, the value 2 obtained the moves by which he can capture other pieces.

So we obtain the series 4,5,6,7. A completely dishomogeneous value, that of Piece 3, remains out.

And we should still determine the move of the last piece, the one that, for exclusion, must be the strongest. Following symbolical reasons¹⁷ linked with the idea of the altar of the fire that stood in the centre of the battlefield and is represented by the square 3x3 at the centre of our magic square,

¹⁴ "The actual procedure of Indian dice game is still shrouded in darkness, owing to the lack of textual sources that describe it. It seems that different versions of the game were used, with "marked" or "unmarked" dice, with and without a board, employing or not employing tokens with the function of marking the spots, which could have played a part in the invention of chess. The game which we are referring to in the epic passages above should be similar to the Vedic name of Vibhudaka, played with walnuts (more than ten of them), which formed a pile; from this pile, following a procedure which is not so clear, a certain number of "dice" would be separated, the amount of which should be divisible by four. When this happened, the player had the best throw, divisible by 4 with 0 remainder, named "ksta", or the "well done". The other throws were called "treat" ("triad"), when the remainder was 3, "dvupara" ("different by two"), when the remainder was 2; and "kali", the worst of all, that had remainder 1. The etymological root of this term is connected with kal-, which covers a semantical area including the concept of "counting" and "constriction, force". And the biggest constrictor, named by Aristoteles "number of the movement from before to after", is precisely kala, Time [...]"

Paolo Magnone, *I dadi e la scacchiera. Visioni indiane del tempo* in *I Quaderni di Avalon*, Rimini, 34 (1995), p. 73-86. (my translation).

¹⁵ David Shulman and Don Handelman, *God inside out*, Oxford university press, New York, 1997. pag. 33
This game, known in the Indian area as cenne, uses a rectangular board with 14 holes. The players have to "sow" the seeds in the holes. The goal is to reach the number of 4. After that the hole will be emptied again. Number 4 represents evidently the reintegration in the original void, this being the symbolical goal of the whole game.

¹⁶ Panaino Antonio, Op. cit. p. 149.

¹⁷ The square 3x3 at the centre of the board represents the tent in the middle of the army camp, where the sacred fire was kept.

Pavle Bidev, *Chess – A mathematical model of the cosmos*, from an article published in *British Chess Magazine*, 1979 – originally in *Mail Chess*, Beograd (Belgrade), December - vol. 1951 and January - vol. 1952 with the title "New Investigations about Chess Origins".

and tracing an analogy with what we did about Piece 3, we can decide that this piece should move and capture in every direction inside this area.

So we obtain:

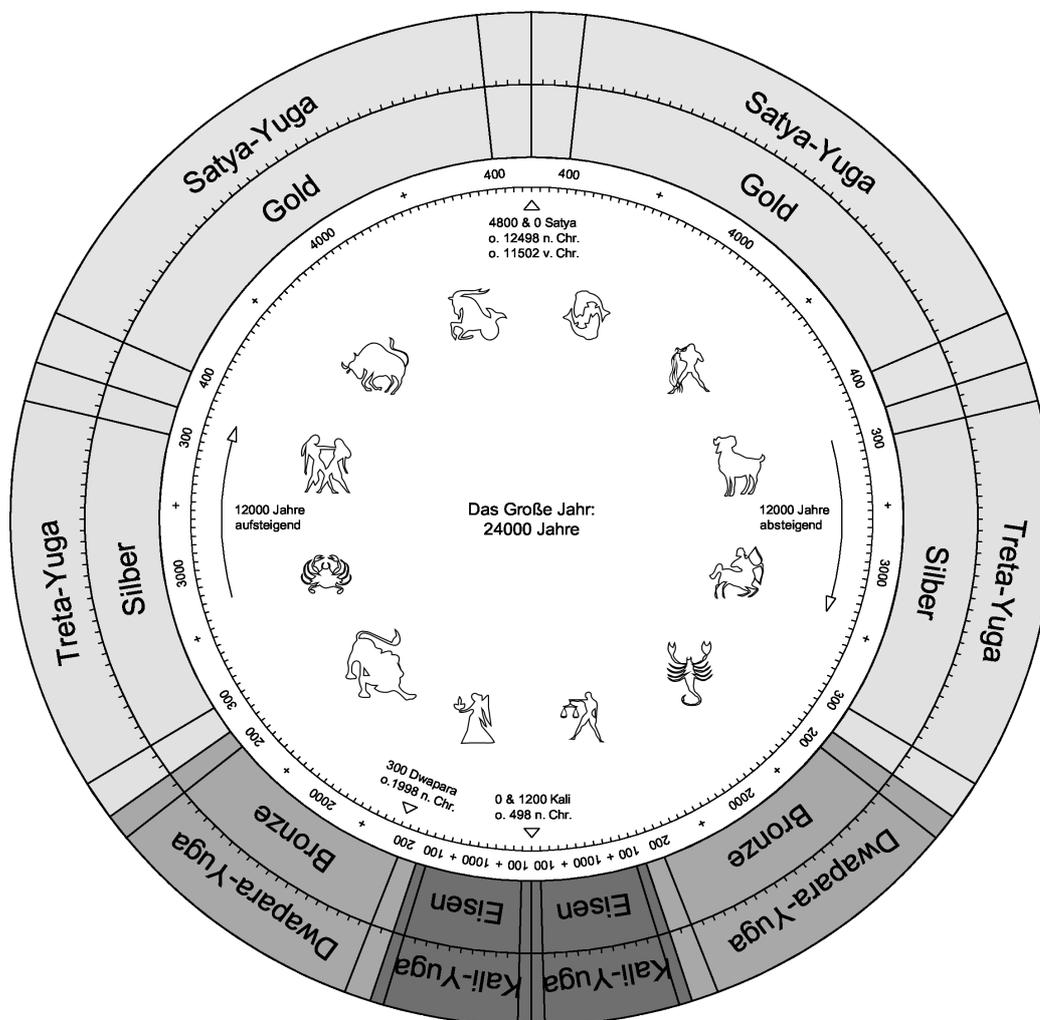
Piece 6: $14 (\text{Piece 3}) - 2 (\text{ordinary Piece 4}) = 12 (\text{King})$

At this point we obtained a series of descending value: 4, 7 (4+3), 6 (4+2), 5 (4+1) and two values outside, one of them referring to the best piece 12 (4 x 3) and the other to the second best 14 (12 + 2).

Confronting the relative moves we obtain the series of the pieces in descending arrangement:

Bishop, Knight, Tower (Rook), Pawn and, on the other side, King and Queen.

Now Wuzurgmihr has determined all the moves and their relation to the pieces following their hierarchical order (probably underlined by the different size of the pieces themselves). It is time to put the pieces on the board, the pawn at the front of the formation, King and Queen at the centre and the others arranged following the decreasing value from the centre to the flank.



The ages of the world (*Yuga*) in Indian cosmology, following Sri Yukteswar. Image by Ingo Kappler.

Critical analysis of the theory

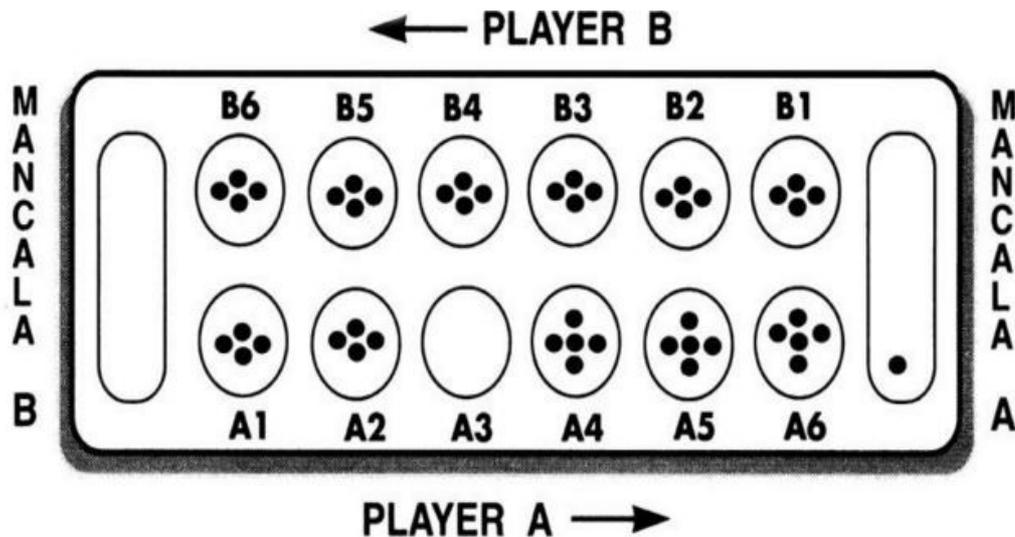
We can expect that the present theory could be judged to be too fantastical. One could argue, for example, that an observer that has no knowledge of the actual moves of chess cannot deduce them in the way we showed. We will respond, first of all, that we don't pretend to provide a unique and final solution. On the other hand we must remember that 275,505,224 different magic squares of order 5 are possible. Even if we believe that the one we used was the one the creators of chess were referring to, we cannot categorically exclude other possibilities. Our interest was not so much in the mathematical demonstration, but in introducing the reader in a way to understand the symbolic and numerical value of the game completely different to that of the contemporary version. We are convinced that the link between the chess board and magic squares is the key to understanding the deep meaning of the tale of Wuzurgmihir. We leave the door open to other scholars, with better mathematical and philological knowledge to verify or falsify this theory.

We will just highlight some points in his favour:

1- The magic square we chose is simple to construct and congruent with the sources of the time. For his mathematical structure, presenting symmetry at the angles, it immediately recalls the moves of the Knight.

2- The presence of a method to count the values based on dice and typically Indian, it is the one based on the number four. As we will see, this is consistent with chess symbology and in particular with its forerunner, the chaturanga. The name itself of this game means "four-sided field" and refers to the division of Indian army during the fourth-third century before Christ, where elephant, chariot, knight and pawn represents the four castes, the elements, the ages of the universe and so on. This symbology leads us to consider the mixed nature of chess, the military and cosmological aspect merging with the one of the racing game, as we can see by the fact that the pawn, at the end of its race, at the eighth square, can be transformed into another piece. This recalls dice and Vedic numeration, as well as pebbles or walnuts used in Vedic times as dice, compatible as well with the game of Mancala.

3- The fact that we are speaking of a symbolic reconstruction, which means that the magic square was not created to visualise the chess moves, but that at least were interpreted and therefore superimposed on the numbers present in the square. This means that a certain grade of deformation is necessary to make the one construct mirror the other. And because it is a myth, it is not reasonable to expect a rigorous mathematical demonstration in the actual sense, or that an observer with no knowledge whatsoever of the game of chess could in fact deduce all moves from the simple observation of the board and the pieces. It is, on the contrary, surprising that the symbolic re-elaborated version can be so near to such an extent.



Scheme of Mancala's board, illustrating the use of walnuts and player's position.
 (From <http://stackoverflow.com/questions/19558417/how-to-abstract-a-mancala-board-in-lisp>)

Encounter and clash of cultures

The WCN, just like the version of this tale provided by Firdawsi (932/42-1020/25), gives a lot of space to the description of the symbolic value of backgammon, the invention of Wuzurgmihr that should defeat the Indian sages. They don't spend a word, however, commenting on chess, merely mentioning that the game should represent a battlefield. Once again we should ask ourselves the reason of missing such a point. Even if we saw the importance of number four in Indian tradition, the WCN and Firdawsi's version seem to imply that the chess presented to Wuzurgmihr was not played with dice. This would highlight the fact that it was a game pointing out the player's ability, leaving randomness outside the board. In other words it represented what we today call "free will". This supposition is confirmed by the enforcement of anti-gambling policies associated with the Hindu renaissance during the Gupta dynasty (sixth century A.C.). This tendency of neo-Hinduism could explain the suppression of dice and the transformation of the racing game of chaturanga into chess in a form more similar to the one we still play today.

Backgammon is presented as the opposite: a game in which fate (through the dice) plays the biggest role. This explains the long focus on the symbolism of the dice and its six faces, as well as its power to move the pawn at its own will. Humans, it is said, are like pawns, which in turn, being 30, represent the days in a month and the degrees of a zodiacal sign, whereas the twelve divisions of the board represent the twelve months in a year and the twelve zodiacal signs. In the same fashion the number seven, which is the constant sum of the opposite sides of the dice, represents the seven astrological planets that weave human affairs.

In other words, the tale wants to present the contest between a game that highlights the capacity of human reason to modify the events and one that seems to insist upon the necessity, for man, to adapt to his destiny. The story coming from a source belonging to the environment of the Sasanian court and imbued with Zoroastrianism, the second attitude is the one being praised. This means that Wuzurgmihr teaches the knowledge of a superior "stoic" and "fatalist" attitude, incarnated in backgammon, in contrast to the one centred on free will that chess seems to suggest.

This world view can seem very strange to a contemporary Western reader, which is on the contrary deep rooted in the second one and for which is difficult even to imagine the first one. The opposite is true for the Islamic culture. This is very interesting, because it could eventually show a return of

the same contrast of opposite worldviews present in our tale.

As-Safadi (1363) speaking about backgammon says: “The points in the game are of advantage or disadvantage of the player following predestination and divine law”. He concludes that this game is therefore superior to chess, because in backgammon the player recognises the power of the fate whereas in the other he denies it¹⁸.

The Indian version of the legendary invention of chess, or at least of its forerunner, Chaturanga, speaks of a wise Brahman, named Sissa, who lived at the court of king Balhait. The king orders the sage to invent a game that shows the utility of reason, of personal judgment and anticipation, as opposed to fatalism. For doing this, Sissa uses the board of side eight and creates pieces that represents the Indian army in its fourfold division and eliminates the use of dice. The king is highly satisfied by this invention, that seems to demonstrate at best the use of reason and free will. He wants to reward Sissa for this accomplishment and asks what his prize should be Sissa simply asks for a grain of wheat for the first square of the board, two for the second, four for the third, and so on, every time doubling the amount of grains. The kings agrees. But then he realise that the quantity of wheat requested is bigger than all the corn present in India ($2^{64} - 1 = 18,446,744,073,709,551,615$). Al-Biruni tells us that this quantity is equal to “2305 mountains” and so “more than in all the world”¹⁹. Another cunning of reason, which, following some versions of the tale (Persian and Arabic) have a bitter end. The king, upset because he cannot pay the promised reward, orders for Sissa to be killed, so at least the arrogance of free will is punished and we have a confirmation of an attitude nearer to fatalism. This comparison between different views of the world and the place of man in the universe is already interesting in itself. But, the nature of chess as a compound game²⁰ and the fact that our tale does not say anything about its symbolism is a clue to investigate deeper in this direction. Maybe we can find under this apparent opposition a common basis between backgammon's and chess's cosmological symbolism.

In other words, a common cultural background of Indian and Persian culture, that in our tale remains hidden by the firework dedicated to backgammon's symbolism, the pride of the Sassanid court and their national genius.



The Panchisi, another possible forerunner of chess. A race game played with dice in form of walnuts (usually 6 or 7, the count is made checking the one that falls on the round or the plate part). The name of the game derives from pachis, a word indicating number 25, the maximum amount of points that can be reached in the game. A possible clue to the link with magic square of order 5. (Image by Daniel Schwen)

¹⁸ Panaino Antonio, Op. cit. p. 209.

¹⁹ Nigel Pennick, *Secret Games of the Gods*, Samuel Weiser Inc., York Beach, 1997. pp. 188- 189.

²⁰ As already said, we will not analyse further the origin of chess, we just assume that elements of different origin were mixed in order to obtain their final form.

Cosmology and symbols

Paul Bidev²¹ has explored in detail the deep connections between chess and the fourfold division of the cosmos. We agree with his analysis.

The most interesting aspect of the matter is that the four pieces that he correlates with the elements are easily found between the stars. In the night sky, or more precisely in the interpretation of star patterns represented by the Indian lunar mansions²².

The knight is the first lunar mansion, Ashvin, which means mare and stretches from 26th degree Aries to 9,20 Taurus²³. Similarly we can find the elephant in Uttara Ashada, whose symbol is an elephant's saddle and occupies from 22,40 Capricorn to 6 Acquarius²⁴. Then we have the Vizier in Magha, the king, that stretches from 26 Leo to 9,20 Virgo²⁵. Lastly, we find the chariot in Vishaka, which actually represents the potter's and cart's wheel, a clear reference to the worker's caste, stretching from 16 to 29,20 Scorpio²⁶.

It is clear from this disposition that such lunar mansions, and so symbolically the chess pieces connected to them, are located to the four cardinal corners of the Zodiac, forming a cross and so assuming the analogy with the four elements, the four seasons, the four castes, and so on.

At this point it is important to recall the doctrine of the four guardians of heaven or the four king stars, a doctrine broadly widespread in Persia and India²⁷, which connected the four cardinal points' (solstices and equinoxes) stars which represented the four cosmic qualities connected in a cross pattern.

To complete the astrological metaphor, we can see the chess piece of the king, corresponding to the immutable element of ether, in the centre of the cross. The pawn, in its role of simple, indistinct token on the board, represents the measure of four seconds of arc, which is the minimum unit of measure resulting from the confrontation of hours and circular degrees. To understand this better, one should think that a circle (in our case the circle of the celestial equator) is divided by 360 degrees. But we divide the day into 24 hours, resulting in a ratio of 1/15 of minute, which is 4 seconds. Precisely this basic unit of 4 seconds is defined in India as "matra" and is identified with the average duration of a breath. A perfect role for our pawn-token on the cosmic board.

But the connections between the game board and astronomy are still not over. In 64 years Venus has 40 cycles²⁸ (and we should remember that, following the reconstruction given above, the 8 pawns should have value 5 and so produce a sum of 40 – Venus has 5 cycles in 8 years). Similarly the magic sum 260 recalls the days in one Venus cycle. And this seems to suggest a time computation based on that number, in order to mediate between solar and lunar cycles using this planet's orbital time.

Furthermore we should note the relation between the board 8x8 and that of 9x9 (not forgetting the

²¹ Bidev Pavel, Op. cit.

²² An Indian lunar mansion or Nakshara (in Devanagari language) is one of 27 or 28 divisions of ecliptic that the moon crosses during its monthly transit. The moon takes 27,3 days to complete its orbit, so it covers approximately one lunar mansion per day.

²³ Simbology confirmed by the corresponding Chinese lunar mansion, the 18th, Mao.

²⁴ The Chinese lunar mansion is the forth, Fang, referring to the sun's chariot and contains Antares and Acab, known in China as Tien Sze, the chariot with four horses.

²⁵ The corresponding Chinese mansion is the 25th, Niao, the Star par excellence, or the seven stars. It contains Alfa Leo or Regulus, which is called by Arabs Malikyy or king star.

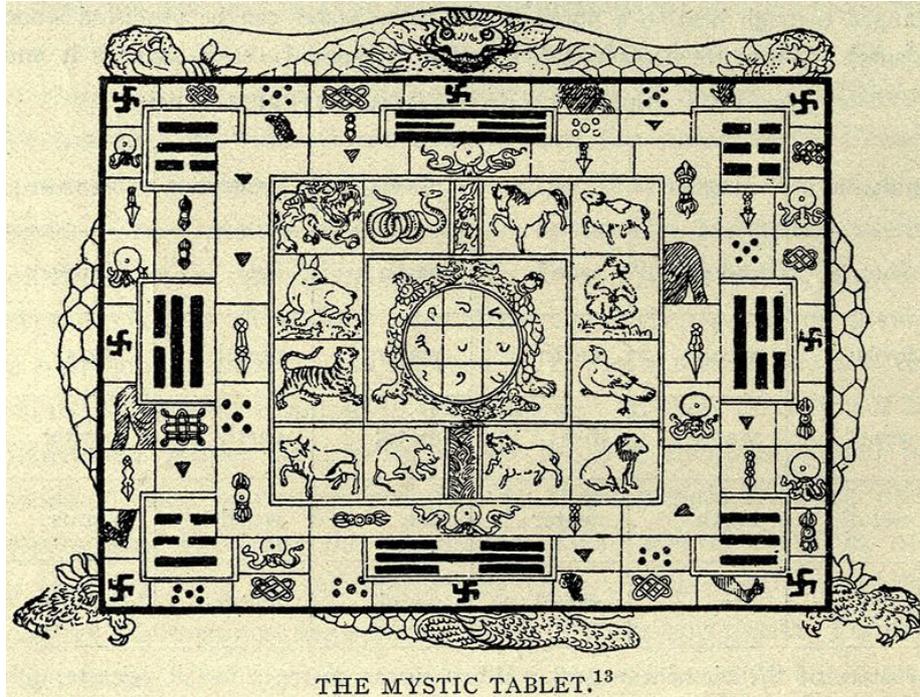
²⁶ The Arab corresponding mansion is the 16th, Al zubana, that describes the claw of a scorpion, two plates of a scale or the yoke. The Chinese mansion 11 Hsu, has the symbol of void and grave.

²⁷ Widespread all over the ancient world. Just think about the transparent connection between the four Evangelists and the four animals of Apocalypse and the linked zodiacal signs, positioned at the four cardinal points of the zodiac (Leo, Taurus, Eagle- Scorpio, Man – Aquarius).

Renzo Baldini, *Trattato tecnico di astrologia*, Hoepli, Milano, 2011. p. 21.

²⁸ Anthony Aveni, *Conversing with the planets: How science and myth invented the cosmos*, Random House, New York, 1992.

one 5x5) we saw above in relation with Indian architecture. Those numbers were very important in the ancient world. Their multiplication produces the cosmic number 360 (8x9x5). The multiplication of the area of the first two by five produces the processional year 259200²⁹. This is the number of years needed by the point of the spring equinox to move around all of the Zodiac. A very important time measure in the old times, closely connected with the idea of the ages of the world.



Correspondences between the square of order 3, the 12 zodiacal signs and the 8 directions (trigrams). Versions also exist where the 28 lunar mansions are also present. (from a Tibetan manuscript cited by Waddell, "The Buddhism of Tibet...", p. 453, and by Carus, " Chinese thought", p 48)

Cosmic music

It is well known that in ancient cultures astronomy, mathematics and music were so close to be almost one thing. It will be no surprise then to find references to musical aspects in chess symbology. The numbers 8.9 and 5, referring to the magic squares immediately recall the value of the tone (9/8) and of the Pythagorean Third (64/81)³⁰.

Again, to understand the meaning of such symbolism we should use our imagination and put ourselves in the shoes of an ancient scholar. Music, astronomy and cosmic order being one and the same, every "disorder" or "discrepancy" in this area was experienced dramatically, like conflicts of consciousness. This being valid for anomalies in the movements of the planets, the comets, the inexplicable events that seemed to destroy the sacral order of the cosmic vault, as well as for mathematical anomalies, like the irrationality of the square root of two (whose secret it is said was punished with death by Pythagoreans) or, even worse, the fact that 6 whole tones were bigger than

²⁹ Titus Burkart, Op. cit.

³⁰ A very important reference to the value 65/64 which Ptolemais provides as an approximation to Pythagorean comma, the difference between 6 whole tones and an octave.

Andrew Barker, *Greek musical writings II: Harmonic and acoustic theory*, Cambridge University Press, 1989. p.299.

the octave and the natural third ($5/4$) was incompatible with Pythagorean scale, which is based solely on multiples of two and three, where the major third ($64/81$) is dissonant.

Other than for the problem of the root square of two, there is a rational solution to harmonise the thirds, the natural one and the Pythagorean one, and it is called comma of Didymus ($80/81$). It is very likely that the ancients saw in this procedure a reflection of the harmonisation of the cycles of the Sun, Moon and Venus (see above).

Anyway, the musicologist Ernest McClain, in his work *Myth of Invariance* proposes a series of correlation between Indian musical scales and castes, correlations that confirm the reconstruction made above.

Indian musical scales are based on the division of the octave in 22 divisions, named shrutis.

Four shrutis made up a whole tone and are linked with the caste of Brahmins. Three shrutis form a minor tone and are linked with the Kshatriyas. Two shrutis form a semitone and are linked with the Vishuddhas. One shruti, which has the value of a comma of Didymus, and is not a real interval, not being used alone, corresponds to a pawn. The vizier and the king would be a Pythagorean third and an augmented fifth. But let us show in detail such correspondences:

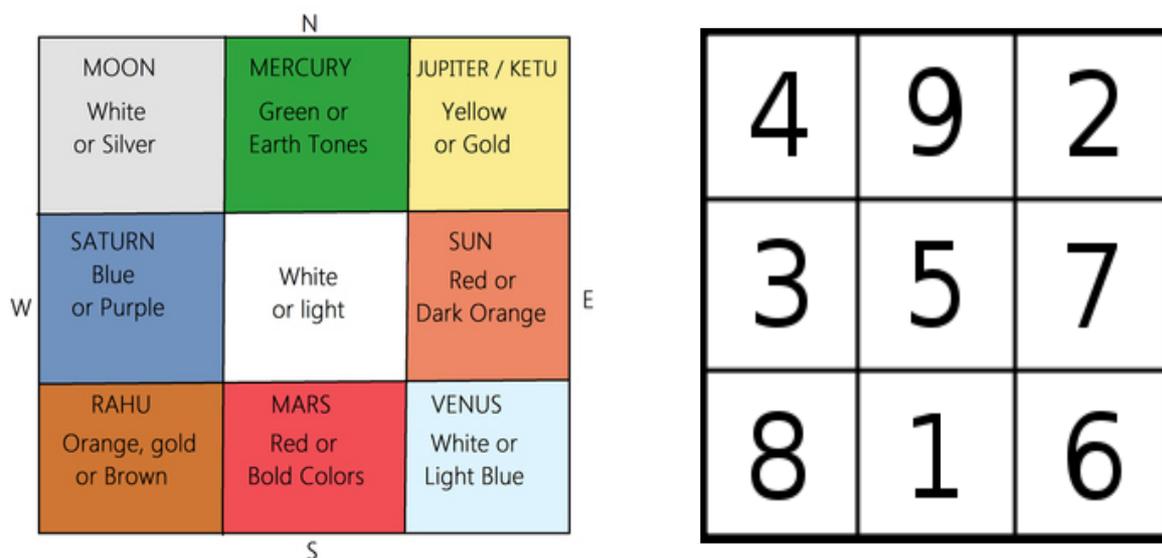
5	4+1	Pawn	Sudra	comma of Didymus	$80/81$
6	4+2	Chariot	Vishuddha	natural semitone	$15/16$
7	4+3	Knight	Kshatriya	natural major tone	$9/10$
4		Elephant	Brahmini	major tone	$8/9$
14		Visir		fifth	$2/3$
12		King		pythagoric third	$64/81$

Alain Danielou, in his *The power of Music*³¹ proposes different correspondences, where the single notes, and not the intervals are linked with the different castes. In this case Sa, Ma and Pa would be Brahmins, Re and Dha Kshatriyas, Ge and Ni Vishuddhas, the alternated notes being correlated with Sudras. There is no contradiction with the correlation provided above, as here the function and not their distance as intervals is considered.

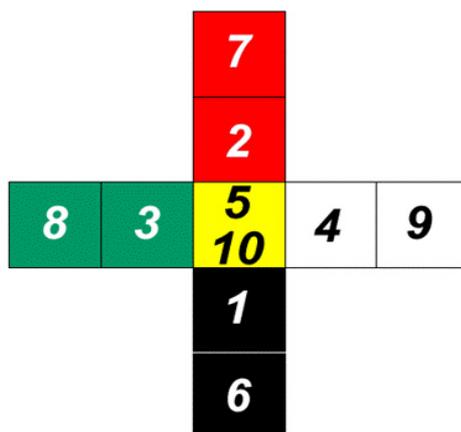
A last daring connection, following Needham's theory, that chess would bear Chinese elements in its own roots, we can embark with Marcel Granet towards the Kingdom of the Middle. If this link could seem at first glance risky, numerical symbology on the contrary confirms a close boundary with this world.

In his *The chinese thought* Marcel Granet shows the numerical values of Chinese pentatonic scales and their connection with cosmological symbolism. This scale is linked in particular with the numbers 5,6,7,8,9 and (10, in the next octave). The connection between the notes, the numbers that represent them and the elements is highlighted by the cross pattern in which they are arranged (which is in turn based on the magic square of order 3, real pivot of ancient Chinese symbolism). Just let have a brief overview of such correlations: in the South we find number 9, linked with fire (Vizier); in the North 6 linked with water (Chariot); in the West 7 correlated to metal (Knight); in the East 8 related to wood (Elephant); at the centre 5/10, the basis note linked to the earth and to pawn/king, because earth, in Chinese thought, is the shapeless element which gives birth to everything.

³¹ Alain Danielou, *Music and the power of the sound*, Inner traditions, Rochester Vermont, 1995. p. 87.



At the right the magic square of order 3 or *Lo shu*, which plays a central role in ancient Chinese cosmology, representing the symbolic link between directions, elements, seasons, and so on. On the left the comparison with the 9 astrological planets in Indian tradition, covering exactly the same pattern. (From <http://www.somamatha.org>)



The *He tu*, a representation of numerical values derived from *Lo Shu*. It is evident the correspondence between the values obtained with our theory and the symbolical associations. (From <http://i-chingcm.blogspot.de>)



Chinese compass, at the centre of a Feng-shui board (*Luo Pan*). We find the same divisions in 4, 8 and 9. The 28 lunar mansions are also present, along with the 12 “earthly branches”.

Last remark: the actual values of this musical scale are obtained multiplying the “simple” number by 8, following the pattern: 40 (5), 48 (6), 56 (7), 64 (8), 72 (9), 81 (10)³². Just note the base value of the scale 40 (8x5 which recalls the relation between the magic square of order 5 and the one of order 8) as well as the last note 81 (that should actually be 80, adjusted by one unit following the method described by Granet), and the presence of 64.

C (Do)	G (Sol)	D (Re)	A (La)	E (Mi)
81	108	72	96	64

The rule for the creation of the musical scale based on the rule of adding and subtracting alternatively 1/3 is very ancient and is attributed to Kuan Tzu (VII century B.C.). The starting point is “take four times three” (81= 3x3x3x3 – maximum Yang) and we proceed applying the rule (linked to Ying-Yang symbolism) and we end up with the exhaustion of the factor 3 when we reach 64 (2x2x2x2x2x2 – maximum Yin). The American musicologist Ernst McClain defines this technique as “the serpent’s tuning” for its zigzag pattern.³³

A common heritage

Now the story has come to an end. Wuzurgmihr also unveiled chess symbolism, which he had omitted before, for a sort of national pride.

What emerges at the end of our path is the existence of a common heritage, which goes beyond the apparent contrast. One heritage which is, however, very different from our contemporary culture. A worldview that felt numerical values almost as living things, with emotions and sensibility. That could see their nuances, their colours their lively relationships. And could find similarities between music, astronomy and symbol with joyful spontaneity. And expressed all this in a game. The board game and its pieces were to these people like huge libraries, containing infinite secrets, but they were different from our libraries, which contain books filled with dead words, because they were libraries created by active imagination, which created symbols and pictures and they could enjoy the smell, the colour and the tone of a concept.

³² As Granet notes the pentatonic scale is created in a descending fashion starting from 81 by descending fourth alternated with ascending fifth, till we reach 64, underlining the value of these two numbers as symbol of “maximum Yang” (odd 3⁴) and “maximum Yin” (even 2⁶).

Marcel Granet, *La Pensee Chinoise: Chinese Thought*, Arno Press, New York 1934.

³³ Jay Kappraff, *Beyond measure, A guided tour through nature, myth and number*, World scientific, Singapore, 2002. pp. 63 – 70.

It is then clear that under the surface of this shallow rivalry between India and Persia, between free will and destiny we find a common way of feeling, a knowledge that unites and connects, because there is not any culture clash, but simply a clash between non-cultures. Because a person of culture is precisely the one that comprehends the other and then eliminates the possibility of a clash. Contraposition can only exist in the absence of culture.

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